Fusion of Distributions for Radar Clutter Modeling

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Abstract – *To recognize an object in an image, an algorithm must identify not only the object pixels, but also non-object clutter pixels. Non-object pixels can be assessed with a priori clutter models that account for the varying terrain and cultural objects. Radar clutter models have been well developed; however, these models typically incorporate a single distribution to capture background effects. In this paper, we propose to use a fusion of distributions using an additive mixture model to characterize various background clutter information so as to more accurately develop a clutter model useful for object recognition. In a radar example, we show a fused-distribution using a Rayleigh and Pareto model describing the average and heavy tail clutter characteristics.*

Keywords: Fusion, Distributions, Clutter Model

1 Introduction

Fusion models [1, 2], such as the ***User Fusion model***, incorporate six levels of information processing. Level 0 includes *a priori* models of the environment and targets, pre-defined mission objectives, and data preprocessing. Level 1 is object assessment. Level 2 and 3 are situation assessment and threat assessment. Level 4 is process refinement and level 5 is User refinement. Since the User defines the location and objects of interest, the User plays a key role in coordinating target recognition. The fusion system must account for known and unknown clutter responses to facilitate timely target recognition. The effective and efficient user-fusion interaction includes clutter modeling of a predefined location.

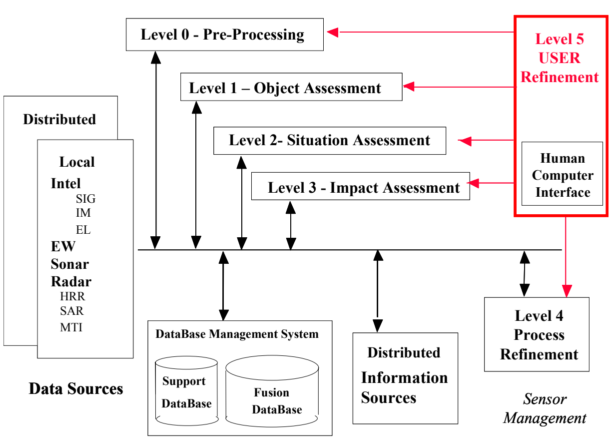

Figure 1. User-Fusion Model

Clutter Modeling is well developed for various radar systems and environmental conditions [3]. While there is a large body of literature on radar analysis, most of the radar parameters are based on empirical application-specific results. Since the application drives the analysis, so does the environment of the defined location. Clutter modeling is based on the uncertainty of the environment as described probabilistically. [4] Thus, the goal of this study is to derive a clutter probability distribution function (PDF) to enhance radar modeling.

Radar modeling typically includes a system signal model which includes a state and an associated variance. [5] Many distributions provide state and variance estimates, and the correct model is one that captures a majority of the data. For instance, a Rayleigh [3, 6, 7], Log-normal [8, 9], or Weibull distribution [10, 11] are used to capture radar clutter. For various background environments (i.e. sea, land, and air) the models have well been developed. Other distributions that have been proposed for describing the statistics of clutter include Ricean, [12] Rice squared, [13] gamma, [14], K-distribution [15, 16], log-normal-Rayleigh [17] and log-Weibull. [18] A single parameter model, in most cases, is not appropriate and further refinement is needed. For instance, at a seashore, the clutter model would have to model both the sea and the land, and if the object of interest transfers from the sea to the land, then the appropriate clutter model would have to be determined. A good example of combining distributions of data is a *mixture model* [17], which utilizes expectation maximization, Metropolis Hastings, or Gibbs sampling to estimate parameters.

A challenging clutter model is a forest. The forest can vary in density and height based on the amount of foliage present. To adequately model the forest clutter, we would like a model that represents both a thick/dense forest as well as one that is sparse/thin. To do this, we propose a fusion of distributions by additive mixture modeling for the analysis which captures the heavy-tail clutter characteristics of the data.

This paper proposes a methodology for in situ estimation of mixture parameters of data using a fusion of distributions and applies the general strategy to clutter modeling. Section 2 details radar target and clutter modeling. Section 3 develops the general theory for the distribution fusion. Section 4 shows a specific example of distribution fusion and Section 5 draws conclusions.

2 Radar Clutter Modeling

Skolnik's text, *Introduction to Radar Systems*, [3] has long been regarded as a key resource for various radar modeling. There are various models types, such as noise, target, and clutter models. Noise factors include: antenna, thermal, and transmission factors. These **radar noise clutter** factors contribute to the false alarms in signal detection.

The radar equation is defined as:[3]

$$R_{max} = \left[\frac{P_t\, G\, A_e\, \sigma}{(4\pi)\, 2\, S_{min}} \right]^{1/4} \qquad (1)$$

where
P_t = transmitted power
G = Antenna Gain
A_e = Antenna effective aperture, m^2
σ = Radar Cross section of the target, m^2
S_{min} = Minimum detectable signal, W

The radar designer can choose all the parameters except for the radar cross section of the target (since these are unknown). Because of (1) noise, (2) uncertainties of target cross section, (3) signal losses, and (4) surface and atmospheric effects; the range must be described probabilistically. Thus, the correct radar range will be a function of the radar return probability distribution function (PDF) (or cumulative distribution function) and probability density function (pdf).

2.1 Probability Distributions

Statistical phenomena can be presented by a PDF, P(x), and a pdf p(x).

$$P(x) = \int_{-\infty}^{x} p(x)\, dx \qquad (2)$$

The benefit of using a probability model is that many systems are effectively described in terms of only two parameters - a mean and a variance. For bandwidth considerations, being able to model the target or clutter with only two parameters is highly desirable.

To model the noise, we use a **Gaussian model**, where the mean (μ) noise is 0, and m_2 is the second moment

$$p(x) = \frac{1}{\sqrt{2\pi\, m_2}} \exp\left(- \frac{x^2}{2\, m_2} \right) \qquad (3)$$

The received signal for radar system with an input noise that is Gaussian can be modeled as a **Rayleigh distribution**.

$$p(x) = \frac{2\,x}{m_2} \exp\left(\frac{-\,x^2}{m_2} \right) \quad , \quad x \ge 0 \qquad (4)$$

Next, we describe standard target models.

2.2 Target Models

Target models are based on σ, the Radar Cross section of the target, or scattering object. σ represents the magnitude of the echo return signal to the radar by the target. The definition of the radar cross section is

$$\sigma = \frac{\dfrac{\text{power reflected toward source}}{\text{solid angle}}}{\text{incident power density} / 4\pi} = 4\,\pi\, R^2 \frac{|\,E_r\,|^2}{|\,E_i\,|^2} \qquad (5)$$

where:

R = range of the target
E_r = electric field strength return
E_i = electric field strength incident on target

Equation 5 assumes that the radar wave is far enough away to be considered planar and that the power is scattered uniformly in all directions. Real targets do not scatter the energy uniformly so we need probability models for the target.

When the wavelength is large compared to the objects dimension, scattering follows a **Rayleigh distribution.** When the wavelength is small compared to the objects dimension, it is in the **optical region**, where the target can be detected. The different regions can be modeled as to the contribution of spheres, rods, plates, cones, and corner reflectors based on associated relationships related to the frequency and wavelength.

Complex target models are based on the magnitude and phase of the signal. The target models of interest are [3]

(1) **Chi square** of degree 2m

$$p(\sigma) = \frac{m}{(m-1)!\, \sigma_{ave}} \left(\frac{m\sigma}{\sigma_{ave}} \right)^{m-1} \exp\left(- \frac{m\, \sigma^2}{\sigma_{ave}} \right) \quad \sigma > 0 \quad (6)$$

(2) **Rice**

$$p(\sigma) = \frac{1+s}{\sigma_{ave}} \exp\left(-\, s - \frac{\sigma}{\sigma_{ave}} (1+s^2) \right) I_0 \left(2 \sqrt{\frac{\sigma}{\sigma_{ave}}\, s(1+s)} \right)$$

where s is the ratio of the radar cross section of the dominant scatterers to the cross section of the small scatterers and I_0 is the modified Bessel function.

and (3) **Log Normal**

$$p(\sigma) = \frac{1}{\sqrt{2\pi\sigma\, s_d}} \exp\left(- \left[\ln \frac{\sigma}{\sigma_m} \right]^2 / \, 2\, s_d \right) \qquad (8)$$

where s_d is the standard deviation of Ln (σ / σ_m) and σ_m is the median of σ. The log normal is used for high target/clutter magnitudes.

2.3 Clutter Models

Radar ***clutter*** is ***unwanted echoes*** from the natural environment, which "clutter" the radar and challenge target detection. Clutter includes echoes from land, sea, weather, and animals. Clutter is spatially distributed and larger in physical size than the radar resolution cell. Man-made objects (i.e. buildings), are "point," or discrete, clutter echoes, that produce large backscatter. Large clutter echoes can mask echoes from desired targets. When clutter is greater than receiver noise, clutter signal processing dominates.

Echoes from land or sea are examples of *surface clutter.* Echoes from rain and chaff are examples of *volume clutter.* The echo magnitude from distributed surface clutter is proportional to the area illuminated. To independently model the clutter echo of the illuminated area, the *clutter cross section per unit area,* σ^0, is utilized:

$$\sigma^0 = \frac{\sigma_c}{A_c} \tag{9}$$

where σ_c is the radar cross section of the clutter occupying an area A_c. σ^0 is a dimensionless quantity, expressed in decibels with a reference value of one, and is also known as the *scattering coefficient, differential scattering cross section, normalized radar reflectivity, backscattering coefficient,* and *normalized radar cross section (NRCS).*

2.3.1 Surface Clutter Radar Equation

In designing surface clutter models, we develop different models based on the grazing angle [19] which can be effective to model land and sea clutter.

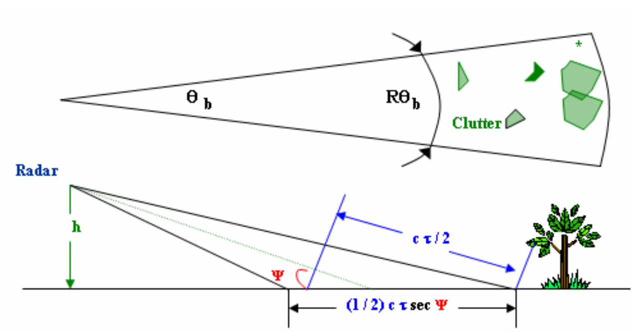

Figure 2 - Elevation geometry shows the extent of the surface illuminated by the radar pulse, (top) plan view showing the illuminated clutter resolution cell consisting of individual, independent scatterers.[3]

Low Grazing Angle. Figure 2 depicts a radar illuminating the surface at a small grazing angle, ψ. For low grazing angles the range is determined by the radar pulse width τ. The cell width in the cross-range dimension is determined by the azimuth beamwidth θ_B and the range *R.* From the simple radar equation, the received echo power P_r is

$$P_r = \frac{P_t\, G\, A_e\, \sigma}{(4\pi)^2 R^4} \tag{10}$$

where P_t = transmitter power, W
G = antenna gain
A_e = antenna effective aperture, m^2
R = range, m
σ = radar cross section of the scatterer, m^2

To model target echos (rather than clutter), we let $P_r = S$ (received target signal power) and $\sigma = \sigma_t$, (target cross section). The signal power returned from a target is then

$$S = \frac{P_t\, G\, A_e\, \sigma_t}{(4\pi)^2 R^4} \tag{11}$$

When the echo is from clutter, the cross section σ becomes $\sigma_c = \sigma^0 A_c$, where the area A_c of the radar resolution cell is

$$A_c = R\, \theta_B\, (c\, \tau / 2) \sec \psi \tag{12}$$

with θ_B = two-way azimuth beamwidth, c = velocity of propagation, τ = pulsewidth, and ψ = grazing angle (defined with respect to the surface tangent). The area A_c in range resolution, is ($c\,\tau$ / 2), where the factor of 2 in the denominator accounts for the two-way propagation of radar. With these definitions, the radar equation for the surface-clutter echo-signal power *C* is

$$C = \frac{P_t\, G\, A_e\, [\theta_B\, (c\, \tau / 2) \sec \psi]}{(4\pi)^2 R^3} \tag{13}$$

When the echo from surface clutter is large compared to receiver noise, the signal-to-clutter ratio is

$$\frac{S}{C} = \frac{\sigma_t}{\sigma^0\ R\, \theta_B\, (c\, \tau / 2) \sec \psi} \tag{14}$$

If the maximum range, R_{max}, corresponds to the minimum discernible signal-to-clutter ratio $(S/C)_{min}$, then the radar equation for the detection of a target in surface clutter at low grazing angle is

$$R_{max} = \frac{\sigma_t}{(S/C)_{min}\, \sigma^0\ \theta_B\, (c\, \tau / 2) \sec \psi} \tag{15}$$

Notice, the range appears as the first power rather than the fourth power as in the usual noise-dominated radar equation, which results in greater variation of the maximum range of a clutter-dominated radar. When there is uncertainty or variability in the range, statistical representations are useful. To model a large grazing angle, we need to address different radar scattering.

Variation of Surface Clutter with Grazing Angle.
Figure 3 shows the general form of surface clutter as a function of grazing angle. There are three different scattering regions. At high grazing angles, the radar echo is due mainly to reflections from clutter that can be represented as individual *planar facets* oriented so that the incident energy is directed back to the radar. The backscatter can be quite large at high grazing angles. At the intermediate grazing angles, scattering is somewhat similar to that from a *rough surface.* At low grazing angles, back scattering is influenced by *shadowing* (masking) and by *multipath propagation.* Shadowing of the trough regions by the crests of waves prevents low-lying scatterers from being illuminated. [3]

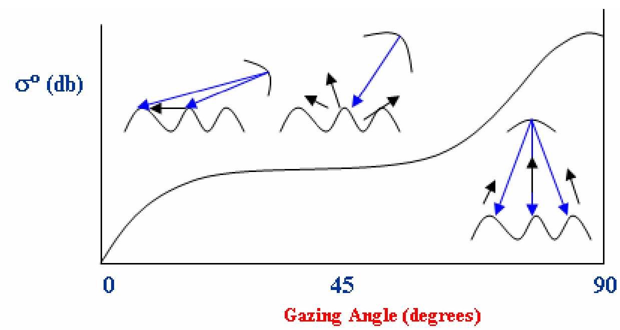

Figure 3 General nature of the variation of surface clutter as a function of grazing angle, showing the three major scattering regions. [3]

High Grazing Angle The surface clutter area viewed by the radar is determined by the antenna beamwidths θ_B and ϕ_B, in the two principal planes. The clutter illuminated area A_C in is $(\pi/4)\ R\ \theta_B\ R\ \phi_B\ /\ 2 \sin\psi$, where ψ = grazing angle and R = range. The factor π / 4 accounts for the elliptical shape of the illuminated area, and the factor of 2 in the denominator is necessary since in this case θ_B and ϕ_B are the one-way beamwidths. Substituting $\sigma = \sigma^0 A_c$, letting $P_r = C$ (the clutter echo power), and taking $G = \pi^2 \theta_B \phi_B$, the clutter radar equation in this case is

$$C = \frac{\pi P_t A_e \sigma^0}{128 R^2 \sin\psi} \qquad (16)$$

The clutter power is seen to vary inversely as the square of the range. This equation applies to the echo power received from the ground by a radar altimeter or the remote sensing radar known as a scatterometer.

Mean and Median Values The characteristics of clutter echoes are usually given in statistical terms. It is often described either by the mean (average) value of σ^0 or the median (the value exceeded 50 percent of the time). For a Rayleigh pdf, the difference between the mean and the median is small (a few percent); but for non-Rayleigh distributions, the mean and the median can be quite different.

For proper radar design, the pdf of the clutter echoes should be known so that the receiver detector can be designed appropriately. Unfortunately, when the clutter statistics cannot be described by the classical Rayleigh pdf, it is often difficult to define a specific quantitative statistical description. For this reason, many radar designs have been based on the mean value of the clutter σ^0 rather than some statistical model. This single parameter model is inadequate in many cases (i.e. forest).

2.3.2 Surface clutter models

Because clutter variability is often described by a pdf or PDF, this section describes several statistical models that have been suggested for characterizing the fluctuations of the surface-clutter cross section per unit area, or σ^0.

Rayleigh Distribution [6] This model is based on the assumption that there are a large number (e.g. 10) of randomly located independent uniform scatterers within the clutter surface area illuminated by the radar.

If the radar receiver uses a linear detector, the probability density function of the voltage envelope of the Rayleigh distributed clutter at the receiver output is

$$p(v) = \frac{2v}{m_2} \exp\left[- v^2 / m_2 \right] \qquad v \geq 0 \qquad (17)$$

where m_2 is the mean-square value (second moment) of the envelope v. The mean m_1 (first moment) is $(m_2\ \pi\ /\ 4)^{1/2}$, and the median is $(m_2 \ \mathrm{Ln}\ 2)^{1/2}$ so that the mean-to-median ratio is $[\pi(4\ \mathrm{Ln}\ 2)]^{1/2} = 1.06$, or 0.27 dB. The mean ($\mu$) of the Rayleigh distribution is proportional to the standard deviation, or

$$\mathrm{StDev} = \sqrt{\frac{4}{\pi} - 1}\ \times\ \mu = 0.523 \times \mu$$

The Rayleigh pdf of v normalized to its median v_m rather than the mean-square value, is

$$p(v_n) = 2\ (\mathrm{Ln}\ 2)\ v_n \exp\left[- (\ln 2)\ v_n^2 \right] \quad v_n \geq 0 \qquad (18)$$

where $v_n = v / v_m$.

The Rayleigh pdf also describes the envelope of the output of the receiver when the input is Gaussian noise. The Gaussian noise assumption applies to the detection of signals in clutter when the clutter statistics are Rayleigh and the clutter echoes are independent pulse to pulse (as is receiver noise). However, this is not often true with clutter. The noise voltage from a receiver is independent from pulse to pulse since it is decorrelated in a time 1 / B, where B = bandwidth. The decorrelation time of clutter can be much longer. This is important when attempting to integrate pulses for improving detection since clutter echoes are generally not decorrelated pulse to pulse. To apply Rayleigh statistics, the clutter might be decorrelated by changing the frequency pulse to pulse or by waiting a sufficiently long time between pulses to allow the temporal decorrelation of the clutter to occur. [3]

Log-Normal Distribution The Rayleigh clutter model applies when the radar resolution cell is large, containing many scatterers, with no one scatterer dominant. It has been used to characterize relatively uniform clutter. However, it is not a good representation of clutter when the resolution cell size and the grazing angle are small. Under these conditions, there is a higher probability of getting large values of clutter (higher "tails") than is given by the Rayleigh model.

One of the first models suggested to describe non-Rayleigh clutter was the log-normal pdf since it has a long tail (when compared to the Rayleigh). In the log-normal pdf the clutter echo power expressed in dB is Gaussian. The log-normal pdf for the echo power when the receiver has a square-law detector is [6]

$$p(P) = \frac{1}{\sqrt{2\pi}\, sP} \exp\left[-\frac{1}{2s^2}\left(\mathrm{Ln}\, \frac{P}{P_m}^{\,2} \right)\right] P \geq 0 \quad (19)$$

where s = standard deviation of *In P,* and P_m = median value of P. The ratio of the mean to the median is exp $(s^2/2)$. With a linear receiver, the pdf for the normalized output voltage amplitude $v_n = v / v_m$ with v_m = median value of v, is

$$p(v_n) = \frac{2}{\sqrt{2\pi}\, s\, v_n} \exp\left[-\frac{2}{s^2} (\mathrm{Ln}\, v_n)^2 \right] \quad v_n \geq 0 \quad (20)$$

where s remains the standard deviation of *In P.*

The log-normal distribution is specified by two parameters (the standard deviation and the median), whereas the Rayleigh is specified by only one parameter (the mean square value). Log-normal clutter is often characterized by the ratio of its mean to median. Based on measurements by different experimenters, the mean-to-median ratio for ground clutter measurement at low grazing angles, it had a value of about 2.6 dB.

It should be expected that because of its two parameters, the log-normal pdf can be made to fit experimental data better than can the one-parameter Rayleigh. However, it is sometimes said that the log-normal tends to predict higher tails for the pdf than normally experienced with most non-Rayleigh clutter, just as the Rayleigh model tends to predict lower values.

Weibull Distribution The Weibull distribution has been applied to land clutter. [10-11] The Weibull distribution is a two-parameter family that can be made to fit clutter measurements that lie between the Rayleigh and the log-normal. The Rayleigh is actually a special case of the Weibull; and with appropriate selection of the distribution's parameters, the Weibull can be made to approach the log-normal.

If v is the voltage amplitude of a linear detector, the Weibull pdf for the normalized amplitude $v_n = v / v_m$ is

$$p(v_n) = \alpha\, (\mathrm{Ln}\, 2)\, v_n^{\alpha - 1} \exp\left[-(\mathrm{Ln}\, 2)\, v_n^{\alpha} \right] \quad v_n \geq 0 \quad (21)$$

where α is a parameter that relates to the skewness of the distribution (sometimes called the *Weibull skewness parameter),* and v_m is the median value of the distribution. When $\alpha = 2$, the Weibull takes the form of the Rayleigh; and when $\alpha = 1$, it is the exponential pdf. The mean-to-median ratio is $(\mathrm{Ln}\, 2)^{-1/\alpha}\, \Gamma(1 + 1/\alpha)$, where $\Gamma(\bullet)$ is the gamma function. With a square-law detector, the Weibull pdf for the power $P = v^2$ is

$$p(P_n) = \beta\, (\mathrm{Ln}\, 2)\, P_n^{\beta - 1} \exp\left[-(\mathrm{Ln}\, 2)\, P_n^{\beta} \right] \quad P_n \geq 0 \quad (22)$$

where $\beta = \alpha / 2$, $P_n = P / P_m$, and $P_m = v_m^2$ is the median value of P.

Application of the Statistical Descriptions of Clutter Caution should be noted when using non-Rayleigh distributions since they are not based on physical properties. They are curve-fitting models used to describe experimental data. Analyses in the literature of the detection of targets in non-Rayleigh clutter can be found that employ Weibull clutter, [11] and log-normal clutter, [9].

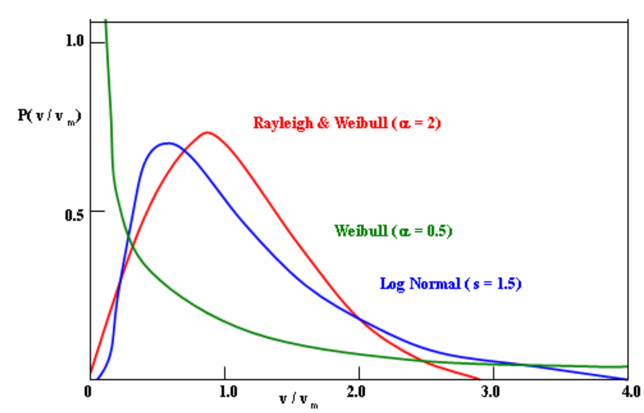

Figure 4 pdfs of the envelope v of the output of a linear detector, normalized by the median value v_m. for Rayleigh, log-normal [with a mean-to-median ratio of 5 dB], and Weibull [with a skewness parameter of 0.5] [3]

A comparison of the Rayleigh pdf with examples of log-normal and Weibull is shown in **Figure 4** (The equivalence of the Rayleigh and the Weibull with $\alpha = 2$ is noted in the figure.) The ability of two parameter distribution like the Weibull to fit experimental data is illustrated by the quite different shapes for the $\alpha = 0.5$ and $\alpha = 2$ curves.

PDFs that contain two parameters, such as the Weibull, can be made to fit experimental data better than can a single-parameter distribution. However, curve fitting of a particular distribution is not always of sufficient help to the radar engineer trying to determine how to design a signal processor to detect signals in non-Rayleigh clutter since a single statistical expression does not do well in fitting the wide variety of real-world clutter.

3 Problem Description

In this paper, we are interested in modeling the clutter observed from an environment with various objects and clutter. The clutter comes from the different natural features in the environment. Since we are interested in the desired targets, we model the rest of the image by establishing a clutter model.

3.1 Problem Statement

The problem we are looking at is modeling forest clutter. There is an associated scene from which we are trying to detect the object and model the various clutter regions (i.e. roads, forest, grass), as shown in **Figure 5**.

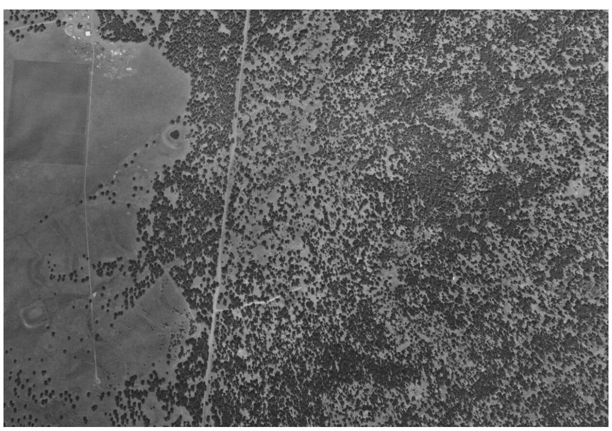

Figure 5. Scene of Interest

For targeting success, and because the ATR tasks have narrow time windows, we need to develop a clutter model for rapid ATR. We seek to model the clutter where the radar power is proportional to the background clutter. By assessing the radar image, we see that there are various land clutter features that contribute to the signal return. To detect a target, we need to effectively determine the background clutter to increase the target-to-clutter ratio. Since a defined distribution for clutter modeling is not effective, we seek a fused clutter model based on a distribution of the clutter histogram.

3.2 Modeling

To model the heavy tails of the spurious clutter, we experienced difficulty with the Rayleigh, Log-Normal and the Weibull for the forest scene. We propose using a Pareto distribution. The distribution of a random variable X is said to be heavy tailed if [20]

$$1 - \mathrm{F}(x) = \Pr[X > x] \sim \frac{1}{\mathrm{x}^{\alpha}} \quad \text{as } x \to \infty, \quad 0 < \alpha \qquad (23)$$

In general, a random variable with a heavy-tailed distribution exhibits a high or even infinite variance and possibly an infinite mean. A random variable with a heavy-tailed distribution will include large values with non-negligible probability. Typically, if such a random variable is sampled, the result will include many relatively small values but a few samples that are relatively large. The Pareto distribution has successfully modeled a wide variety of phenomena from the social and physical sciences to communication theory. The **Pareto distribution** parameters k and α (k, $\alpha > 0$), can be represented with density and distribution functions

$$\mathrm{f}(x) = \mathrm{F}(x) = 0 \qquad (x \leq k) \qquad (24)$$

$$\mathrm{f}(x) = \frac{\alpha}{\mathrm{k}}\left(\frac{\mathrm{k}}{\mathrm{x}}\right)^{\alpha+1} \qquad (25)$$

$$\mathrm{F}(x) = 1 - \left(\frac{\mathrm{k}}{\mathrm{x}}\right)^{\alpha} \qquad (x > k;\ \alpha > 0) \qquad (26)$$

and a mean value

$$\mathrm{E}\,[\,X\,] = \frac{\alpha}{\alpha - 1}\mathrm{k} \qquad (\alpha > 1) \qquad (27)$$

The parameter k specifies the minimum value of the random variable and α determines the mean and variance of the random variable: If $\alpha \leq 2$, then the distribution has infinite variance, and if $\alpha \leq 1$, it has infinite mean and variance. **Figure 6** compares the Pareto and exponential density functions on a log-linear scale.

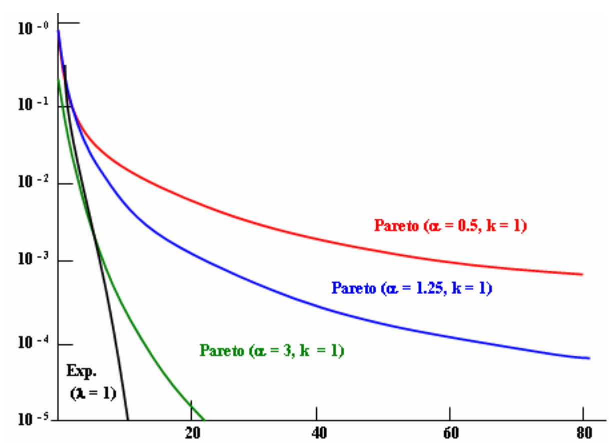

Figure 6. Pareto and exponential PDFs.

Note that on this scale the exponential density function is a straight line, reflecting the exponential decay of the distribution. The tail of the Pareto distribution decays much more slowly than exponential; hence the term ***heavy tail.***

3.3 Fusion of Distributions

The fused distribution approach requires some thought associated with model assumptions. In most distributions, there are associated moments and thus the combined moment is different than the individual moments.

We combine the (1) Rayleigh and the Pareto distributions in this study for the clutter and the (1) Rayleigh-Log-Normal, and (3) Rayleigh-Weibull are not shown. Assuming we have a distribution $f_i(x)$ where i = 1, …, n distributions, we can weight the distributions

$$f_F(x) = a_1 f_1(x) + \ldots + a_n f_n(x) \tag{28}$$

based on the contribution of the distribution to the reduce the error between the fused distribution $f_F(x)$ and the normalized histogram clutter. The weights are normalized values, where

$$a_i = a_i \Big/ \sum_{i=1}^{N} a_i \tag{29}$$

Another method to combine distributions, is to determine the effective range over which one model is more accurate than another. Without extensive evaluation of all the land clutter, this would prove ineffective. Thus, we fuse the distributions to model the average clutter region.

The statistics, such as the mean and variance, of the fused model are important for clutter modeling and target detection. For the mean and variance, we weight (in the additive fusion case) them below:

$$\mu_F = a_1 \mu_1 + \ldots + a_N \mu_N \tag{30}$$

$$\frac{1}{\sigma_F^2} = a_1 \frac{1}{\sigma_1^2} + \ldots + a_N \frac{1}{\sigma_N^2} \tag{30}$$

The resulting statistical parameters can be used to model the clutter for land surfaces.

4 Results

For the results, we present the standard clutter models (i.e. Rayleigh) followed by the fused distribution model. For the analysis, we use both the histogram analysis and a quantile-quantile (Q-Q) plot. The Q-Q plot can provide more insight into the nature of the statistical difference than analytical methods such as the chi-square and Kolmogorov-Smirnov 2-sample tests.

The histogram analysis shown in **Figure 7** allows us to model the clutter in the image and develop the underlying PDF.

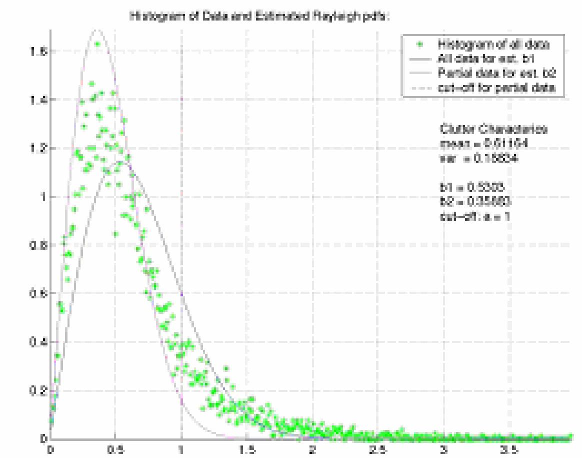

Figure 7. Scene of Interest

The quantile-quantile (Q-Q) plot is a graphical technique to determine if two data sets (of equal or non-equal size) come from populations with a common distribution. A Q-Q plot is a plot of the quantiles of the first data set against the quantiles of the second data set. A 45-degree reference line is also plotted. If the two sets come from a population with the same distribution, the points should fall approximately along this reference line. The greater the departure from this reference line, the greater the evidence for the conclusion that the two data sets have come from populations with different distributions. To determine if the reference image is the same as the measured image, we show the Q-Q plot. **Figure 8** shows that the Rayleigh model suffers from inadequately modeling the tail of the distribution.

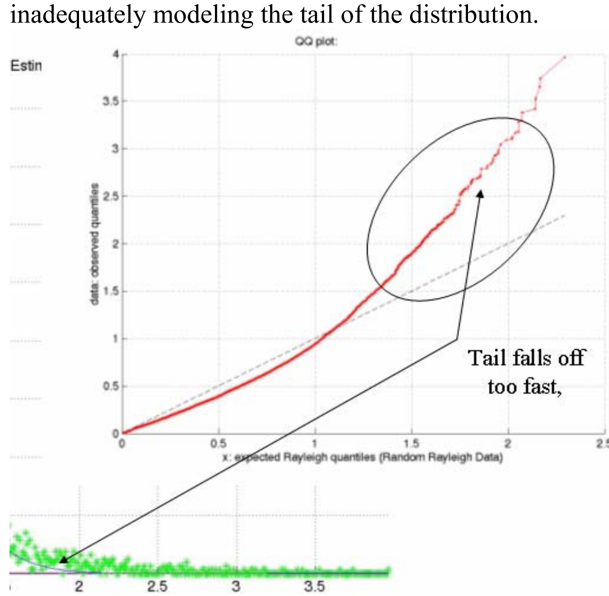

Figure 8. Heavy tail deviation

Using the fused distribution approach, we determine that we could model the tail and the general clutter. Determining the threshold for the breakpoint of the clutter region and the corresponding tail would be dependent on the scene. Thus we chose to fuse the two distributions to obtain a combined two-parameter statistical model. **Figure 9** and Figure 10 show the desired fused distribution for the image in question.

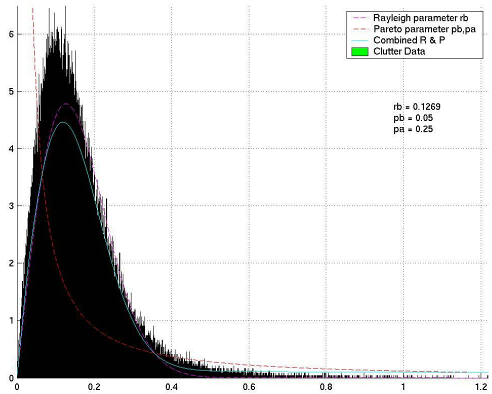

Figure 9. Scene of Interest

Figure 10 shows the results of the distributions, without the clutter histogram. The fused distribution in Figure 10 shows the benefits of fusing the data as opposed to just determining a threshold for using independent models. The resulting error between the PDFs is smaller, and we can determine the mean and variance of the clutter based on the image content. Furthermore, we addressed the corresponding features in the image to weight the density of clutter objects to the model.

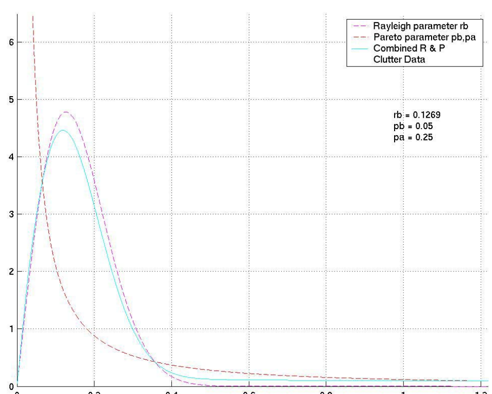

Figure 10. Fused Distributions

5 Conclusions

In this paper, we presented a method for fusing different distributions and applied the methodology for characterizing the clutter associated with radar analysis. The method of fused distributions by additive mixture modeling can be applied to different systems so as to effectively determine a statistical nature of the system. Using the example presented, future research includes more extensive comparisons with the expectation-maximization algorithm in capturing the heavy-tail data and showing the distribution modeling gain for target detection.

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